Non-linear system:

Solve economic equilibrium using non-linear equation.

Solve for competitive equation.

Write FOC, and solve that find equilibrium.

Might be hard to solve because of condition.

How do we solve it?

Matrix of partial derivative is invertible at solution. Otherwise, in trouble.

Newton’s method:

Have a good guess or start grid of guess

If f is continuous

Python has fsolve

Newton Hybrid: solve system and how well it does

Method = f.minimize()

IPOPT: can solve large sparse system, has Python wrapper. Need C++ compiler.

Will use this method for Dr. Schwidiger.

Python package specialized for constrain minimization. Function 0 with system of equatios.

Gradient steepest descent

Tensorflow

Minimizing quadratic functions

Stochastic Ramsey Model:

1 guy in the model.

Maximized expected utility.

Already Planners problem.

Choose how much consume and invest. L is fixed.

Provide inelastic labor.

How much to eat and invest.

Shocks on depreciation rate.

Assumptions on uncertainty to solve numerically.

Numerical integration

Solve consumption and investment for every period = solve problem.

But this would take too much time, so solve policy function.

There exists stationary solution require theory.

How to represent function using function approximation.

For, one dimension, discretize. 10, 10.5, 11, …

Easy to find one point of function, which is stead state. Use Taylor’s rule to find linear approximation to policy function. Linear approximation not horrible, if not far away from the steady state.

Markov Control Processes.

Principal optimality.

State space could be utilitally space.

Each state, assign action.

If you know v function, you are good.

Solution as value function. For each x can get policy function by finding the solution to simple non-constraint optimization problem.

Existence of solution. Proof existence of solution by contraction mapping theorem.

Assumption on transition matrix and prove existence by monotone convergence theorem.

If assume X and A are finite. Finite dimensional problem so iterate and sequence cluster and show existence of fixed point.

Trouble if X is continuous. Then, need to consider space and function to find solution.

Continuous function good for approximation and bounded.

Board games. State space is large by finite. Moves are finite. Finite problem. Can you solve this with dynamic programming? If can, irrelevant. For every position of board can find winning strategy if solve policy function. Stochastic dynamic programming that does approximation.

Ramsey model is not finite problem. Idealize with everything continuous. K is naturally infinite. Discretize and make it a finite problem. How can we approximate to infinite problem?

Easier to solve if make assumptions:

1. Transition problem …

X is finite, but action is infinite.

Log utility not bounded below. At end of day, irrelevant because can bound everything and zero is no longer a problem.

Consumption can go to zero is bad because of negative infinite marginal utility.

So, log utility until 10^-10 and after that change utility function etc.

Discount ensures that there exists a solution. For finite case, there is an optimal solution. For each state x associate action A(x) and search for optimal value of function, say minimal. We can use value function iteration.

Beta affect speed of convergence in FDP.

Interest rate low near 0, so beta better be high like 0.99

With low beta should converge fast, and should converge for 0.99 as well.

Bellman operator is on infinite dimensional space, so how do you operate it? Use a polynomial function that matches with finite many points.

But with unknown function, need to do integration and not summation.